A Random Walk on Stochastic Quantum Dynamics

for (and by) amateurs…

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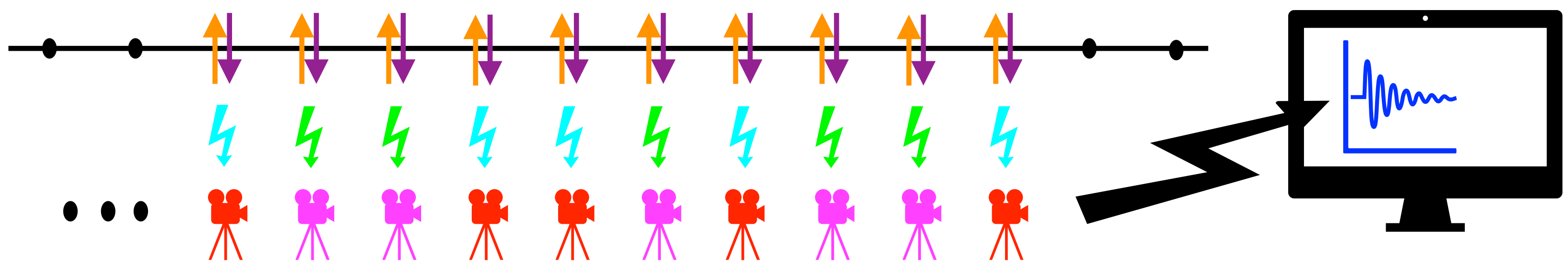
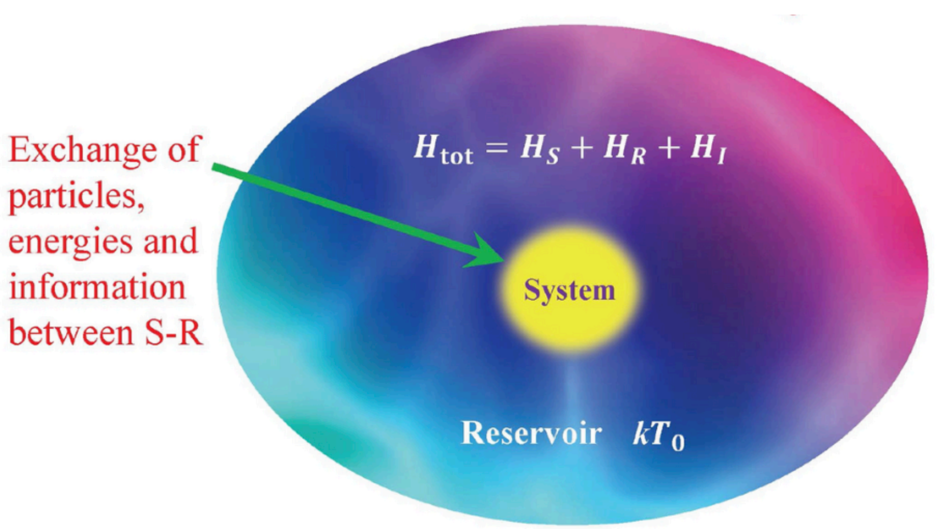
Stochasticity/Randomness in Quantum Mechanics

—> « **Intrinsic** », eg. via monitoring (with application to control)

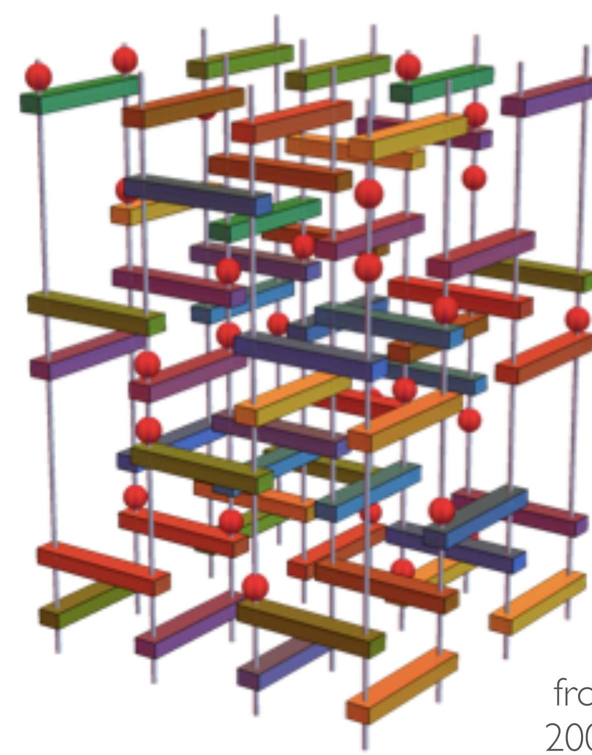


—> « **Extrinsic** », eg via interaction with environments (in or out-of equilibrium)



—> « ***Generic*** », eg to extract generic properties

random matrices (quantum chaos)
random quantum circuits



from arXiv 2009.11311

Balades around Quantum Stochastic Processes : Menu

I- « Intrinsic » : Measurement and quantum trajectories

indirect measurement and quantum trajectories
open system dynamics and CP map (Lindblad dynamics)
excursion in probability theory (martingale, Bayesian update,….)

II- « Generic » : Entanglement & measurement in many-body systems

measure/test of entanglement in many-body systems
modeling generic chaotic systems
random quantum circuits

III- « Extrinsic » : Transport & fluctuations in mesoscopic systems & QSSEP

noisy/stochastic quantum dynamics (versus open quantum dynamics)
quantum diffusive systems
(classical) macroscopic fluctuations theory
excursion in probability theory (large deviation theory, Brownian motion & SDE…)

Balades around Quantum Stochastic Processes

I- « Intrinsic » : Measurement and quantum trajectories

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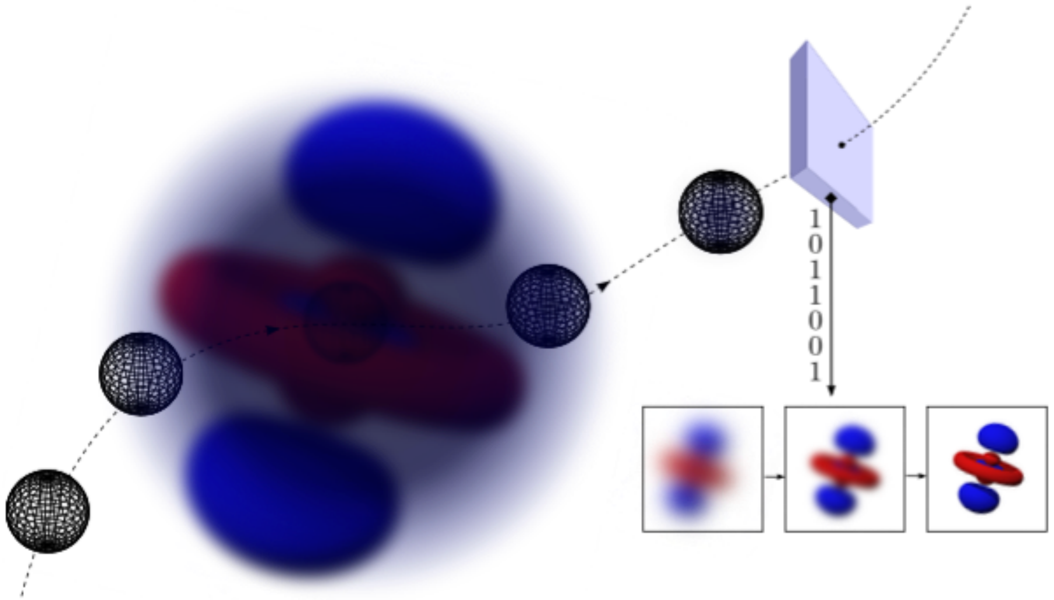
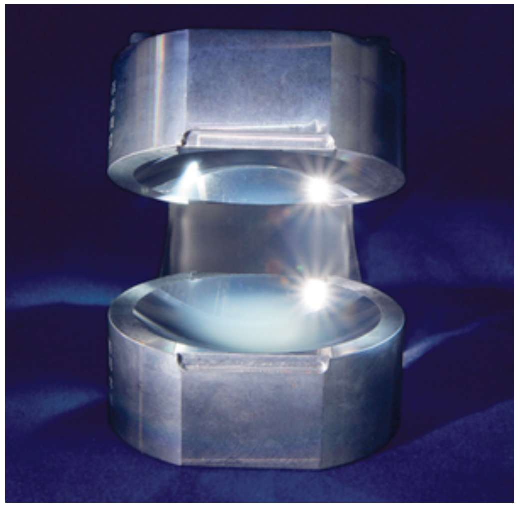


Haroche's experiment & the « photon box

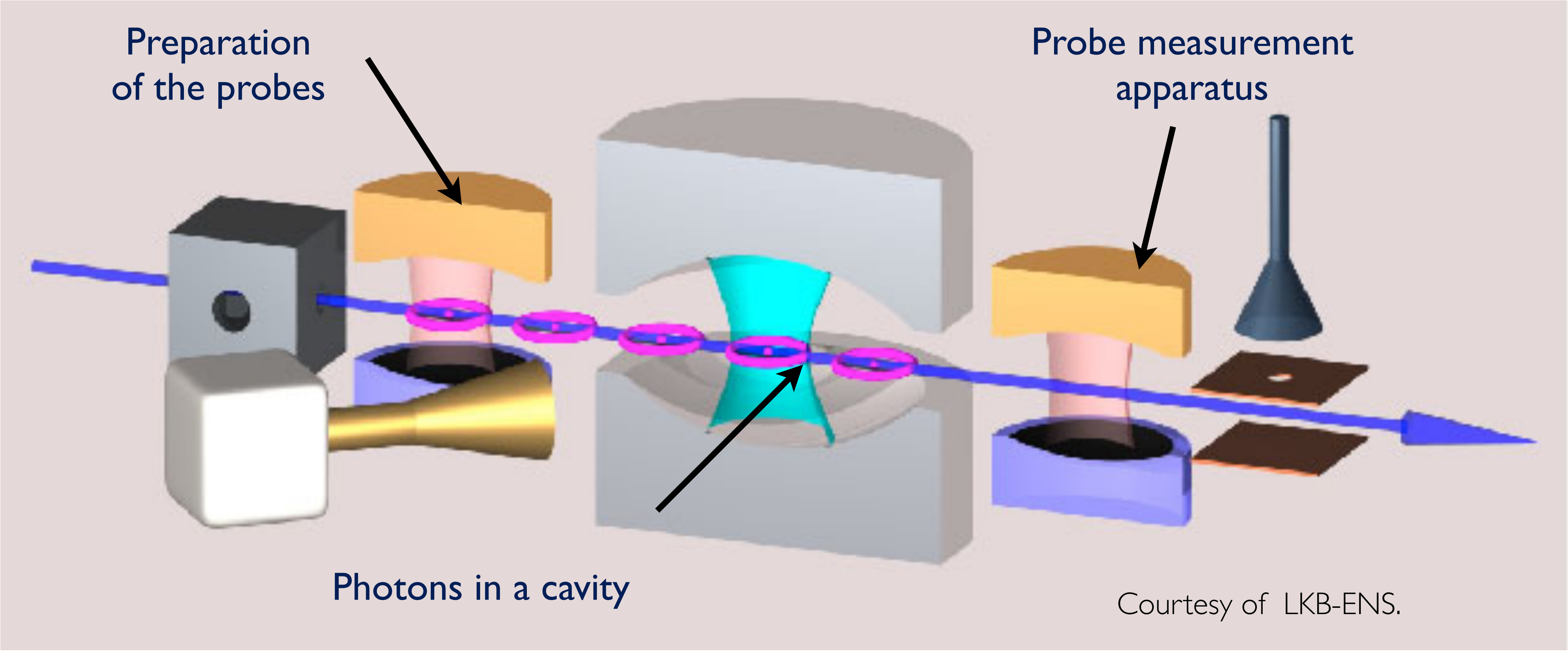


Indirect measurements + entanglement => partial information

System (S) = photons in a cavity.
Probes (P) = Rydberg atoms (two state systems)

« Haroche's photon box »

(Nobel 2012)



Effective rotation of the probe effective spins.

$$U = \exp\left[i\,\theta\, N_{\rm photon}\, \sigma^z \right]$$

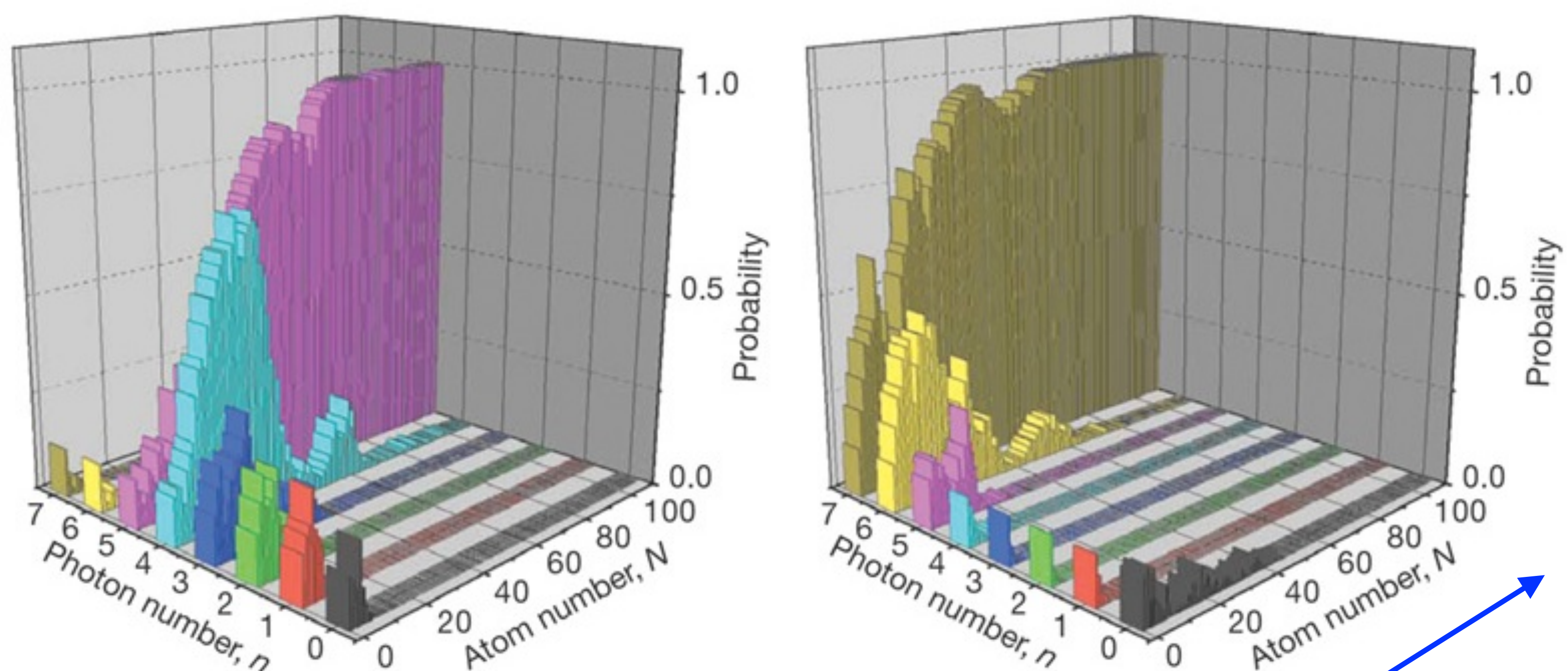
Non-demolition measurement and progressive collapse

Progressive field-state collapse and quantum non-demolition photon counting

Christine Guerlin[1], Julien Bernu[1], Samuel Deléglise[1], Clément Sayrin[1], Sébastien Gleyzes[1], Stefan Kuhr[1]†, Michel Brune[1], Jean-Michel Raimond[1] & Serge Haroche[1,2]

Figure 2 | Progressive collapse of field into photon number state.

. **c,** Photon number probabilities plotted versus photon and atom numbers n and N. The histograms evolve, as N increases from 0 to 110, from a flat distribution into $n = 5$ and $n = 7$ peaks.

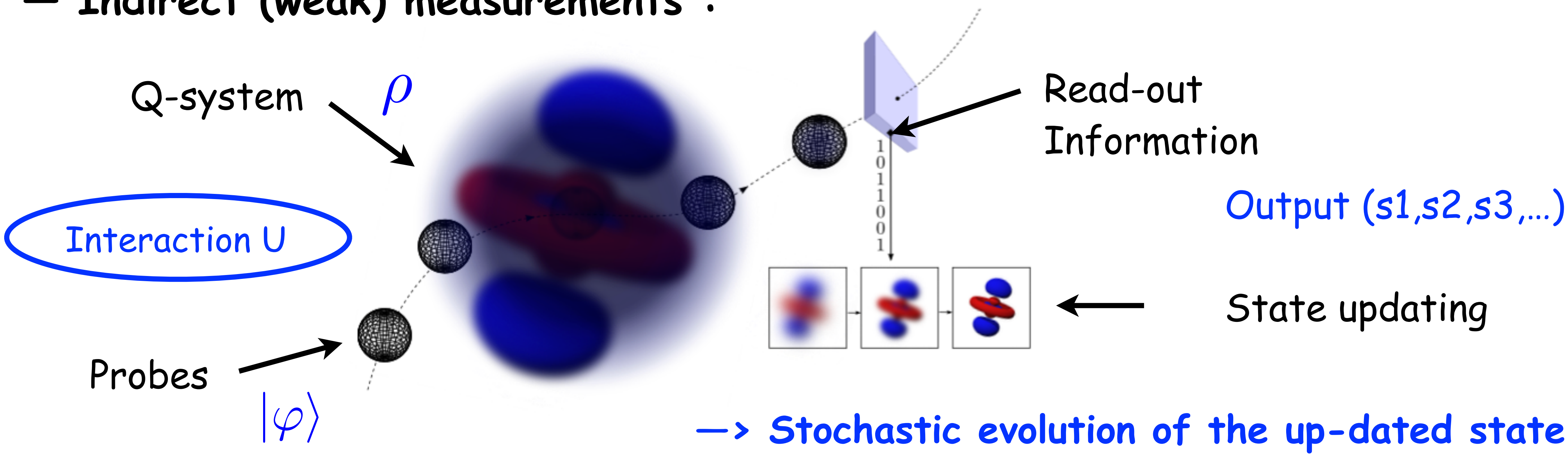


P(n) = Probability distribution of the number n of photons in the cavity

Number N of atoms which have been measured after crossing the cavity

Indirect measurements and Quantum trajectories.

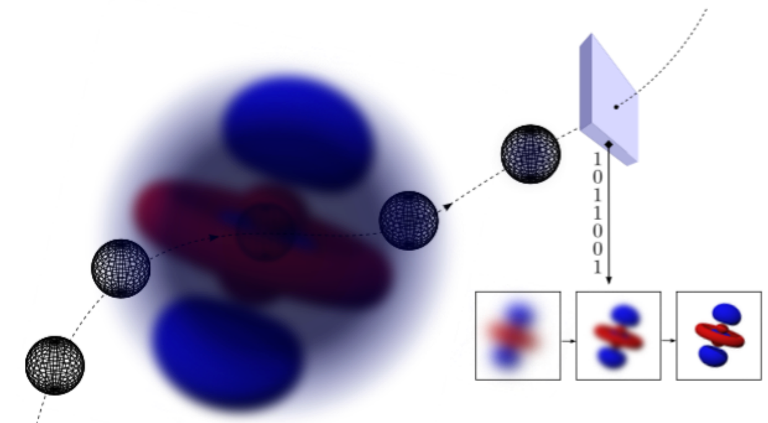
— **Indirect (weak) measurements :**



—> **Stochastic evolution of the up-dated state**

— **« Quantum trajectories » :** —> cf. blackboard...

Indirect measurements and Quantum trajectories.



— **Indirect (weak) measurements (… / see above)**

— **« Quantum trajectories » :**

[Carmichael, Caves-Milburn, Castin-Dalibard-Molner, Zoller,….]

Information from probe measurements : $(s_1, \cdots, s_n, \cdots)$

System state up-dating :

$$\rho \to \frac{F_s\, \rho\, F_s^\dagger}{\pi(s)} \quad \text{with probability} \quad \pi(s) = \mathrm{Tr}(F_s\, \rho\, F_s^\dagger)$$

(random because of Q.M.)

with $\sum_s F_s^\dagger\, F_s = \mathbb{I}$ (POVM)

—> Application e.g. to monitoring or quantum control.

— **Mean behavior & CP-map (Q-channel) :** —> cf. blackboard…

Infinitesimal form: Lindblad dynamics

Back to non-demolition measurement and progressive collapses

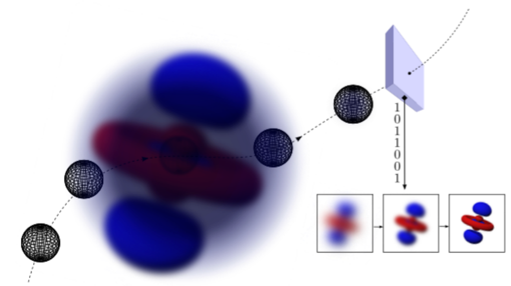
— **Non-demolition measurements:**

Interaction preserves a basis of system states, alias « pointer states »:

$$U = \sum_k |k\rangle\langle k| \otimes U_k$$

pointer states ↗

acting on probes ↖



State up-dating : $\rho \to \dfrac{F_s\, \rho\, F_s^\dagger}{\pi(s)}$ —> cf. blackboard…

— **Random evolution of the diagonal matrix elements**

of the system density matrix (in the pointer basis):

$$Q_n(k) := \langle k|\rho_n|k\rangle \to Q_{n+1}(k) = \frac{p(s|k)Q_n(k)}{\pi_n(s)}$$

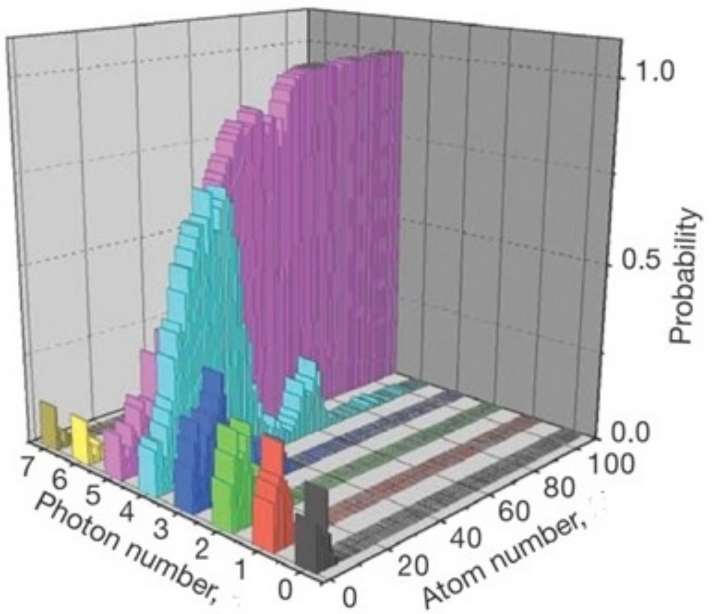
with probability $\pi_n(s)$

—> Bayesian update (from Q.M.)

Non-demolition measurement and progressive collapses

— **Random evolution of the diagonal matrix elements** of the system density matrix (in the pointer basis):

$$Q_n(k) := \langle k|\rho_n|k\rangle \to Q_{n+1}(k) = \frac{p(s|k)Q_n(k)}{\pi_n(s)}$$ with proba $\pi_n(s)$



—> **Martingale** property & martingale convergence theorem

—> cf. blackboard…

— « **Convergence/Progressive collapse** »:

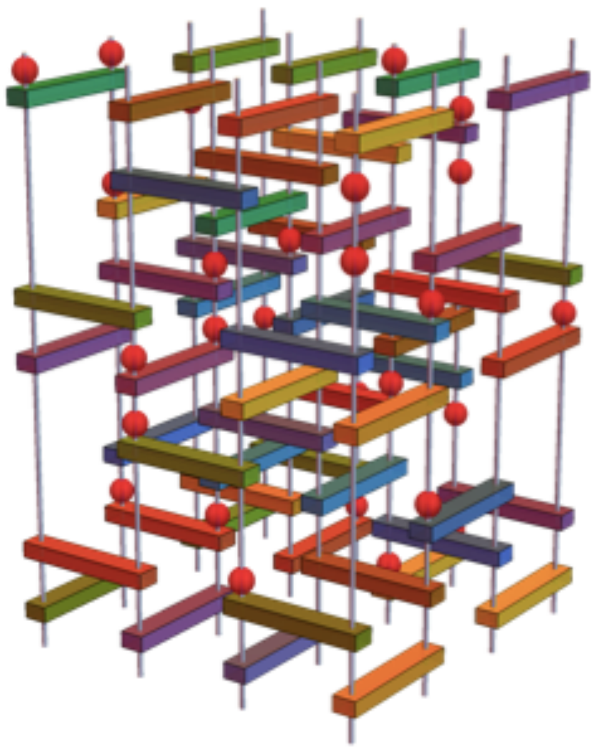
- The sequences $n \to Q_n(k)$ converge for any k (almost surely and in L1).
- The limit distribution is peaked: $Q_\infty(k) = \lim_{n\to\infty} Q_n(k) = \delta_{k=k_\infty}$
- The target is distributed according to the initial distribution: $\mathbb{P}[k_\infty = p] = Q_0(p)$

—> « Mesoscopic measurement apparatus » & Quantum-to-Classical transition.

Balades around Quantum Stochastic Processes

II- « Generic » : Entanglement & Measurement in many-body systems

measure/test of entanglement in many-body systems
modeling generic chaotic systems
random quantum circuits

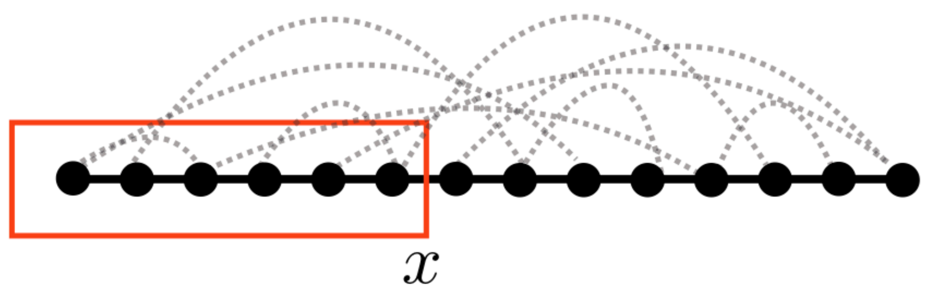


— **Random Quantum Circuits**
(Cf. A Nahum et al, + many papers)

Generic, unstructured models of local many-body quantum dynamics

Entanglement growth in 'generic, chaotic' many-body quantum systems

—> Unitary dynamics generate entanglement.



(as time increase: linear growth up to saturation...)

... quantified via Renyi entropies

$$S_n(x,t) := \frac{1}{1-n} \log \mathrm{Tr} \rho^n(x,t)$$

with $\rho(x,t) := \mathrm{Tr}_{x<}\, \rho_{\mathrm{sys}}(t)$

— As time increase: linear growth up to saturation...(up to fluctuations)

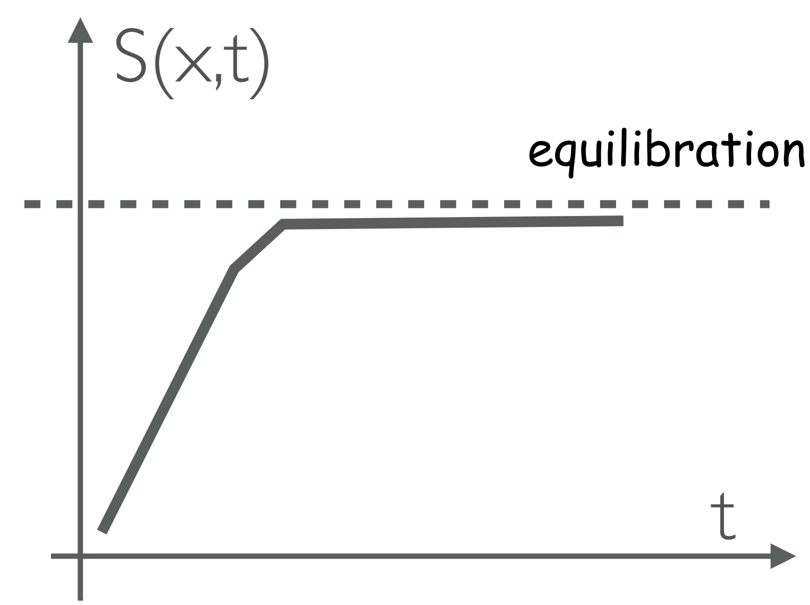
Starting from a product state (finite volume) :

- linear growth $S(x,t) \simeq v_e t \quad (t \leq t_{eq})$

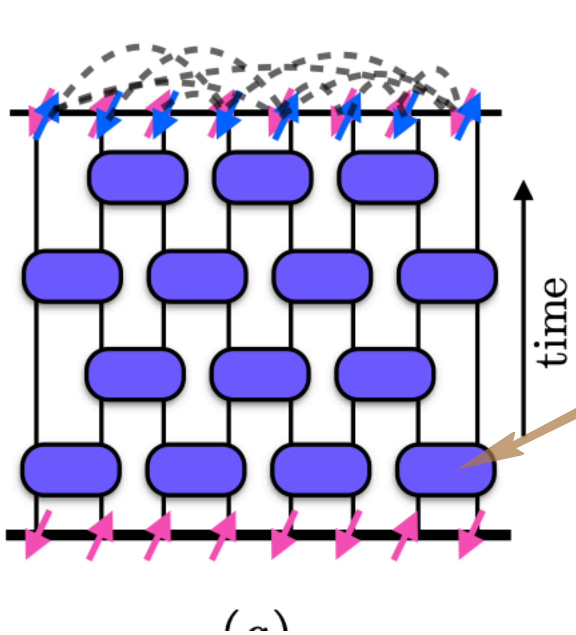
- saturation $S(x,t) \simeq s_{eq} x, \quad (t \geq t_{eq})$

(equilibration / volume law)

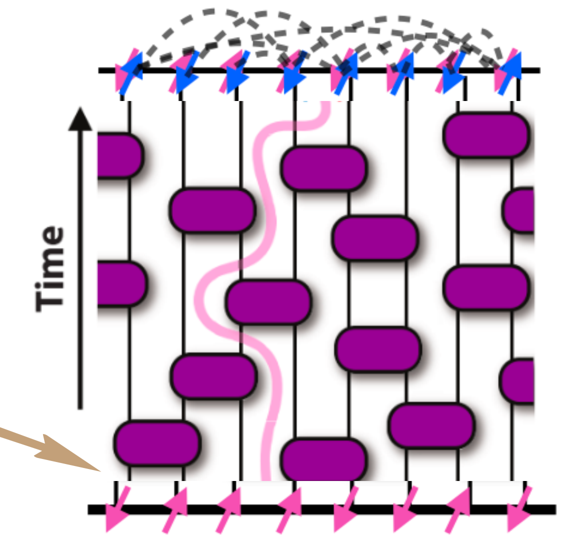
(size of [0,x] = x)



—> Modeling via random unitary circuits [with minimal structure (locality in space & time)]



« 2-site gates »
Haar distributed



(Cf. A Nahum et al, + many papers)

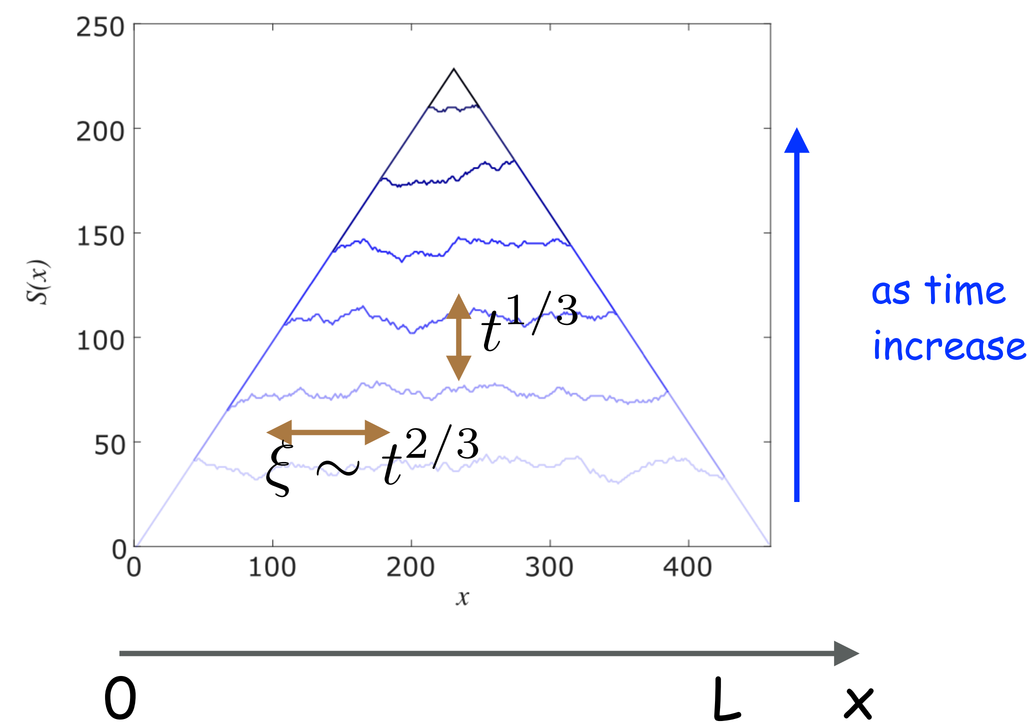
— **Fluctuations** : (... à la KPZ, for the spare random RQC)

- linear growth $S(x,t) \simeq v_e t \quad (t \leq t_{eq})$

- saturation $S(x,t) \simeq s_{eq} x, \quad (t \geq t_{eq})$

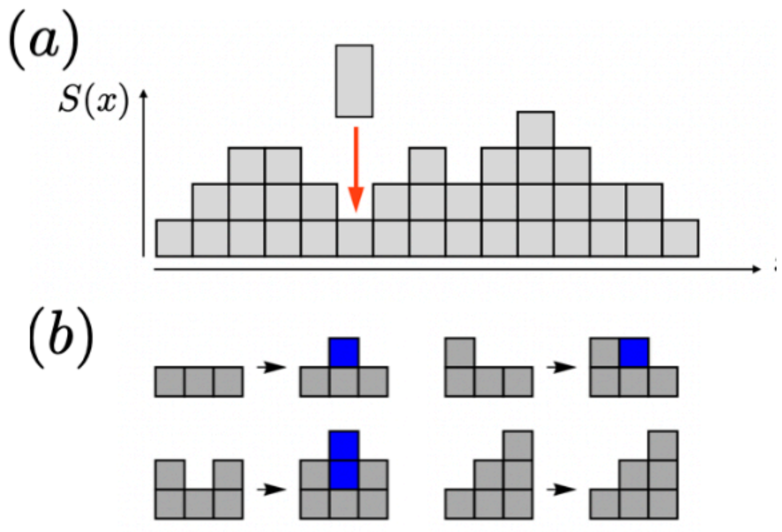
(equilibration / volume law)

(size of [0,x] = x)



— **Fluctuations :** (… à la KPZ, for the spare random RQC)

$$S(x,t+1) \simeq \min[S(x-1,t), S(x+1,t)] + 1$$



maximization of the sub-addivity bound

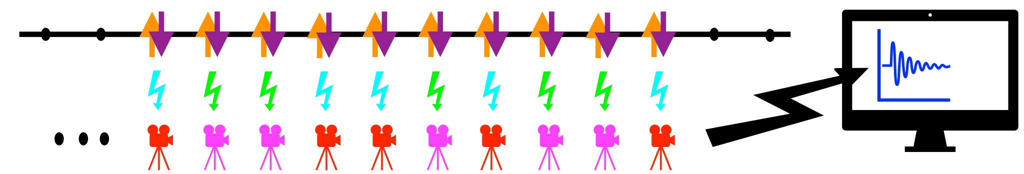
$$|S(x+1,t) - S(x,t)| \leq 1$$

Haar random (n large) —> cf. blackboard…
(if needed.. if time permits)…

$$\overline{n^{-S_2(x,t+1)}} = \frac{n}{n^2+1}\left[\, \overline{n^{-S_2(x+1,t)}} + \overline{n^{-S_2(x-1,t)}} \,\right]$$

Quantum Monitoring in Many-Body Systems

— **Mean behavior** (-> open system -> transport)



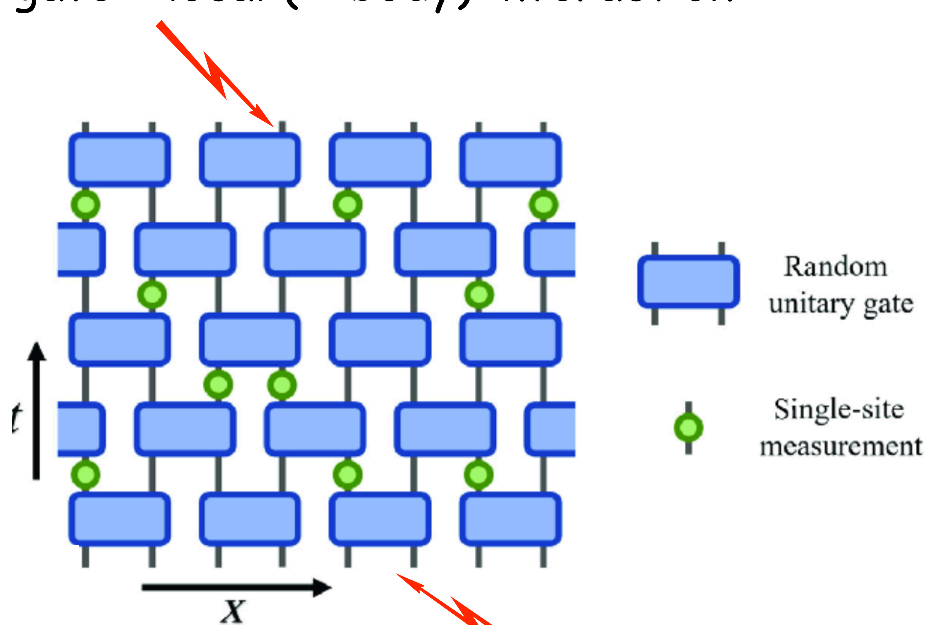
— **Competition between**

- — Entangling unitary dynamics
- — Desentangling (weak/projective) measurements

—> Measurement induced phase transition

— **Modeling via random unitary circuits**

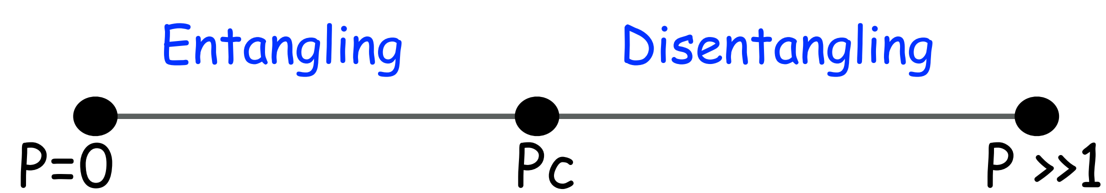
gate = local (2-body) interaction



local spins $\mathbb{C}^n$

(Cf. A Nahum et al, + many papers)

« MiTP »



P = Monitoring rate

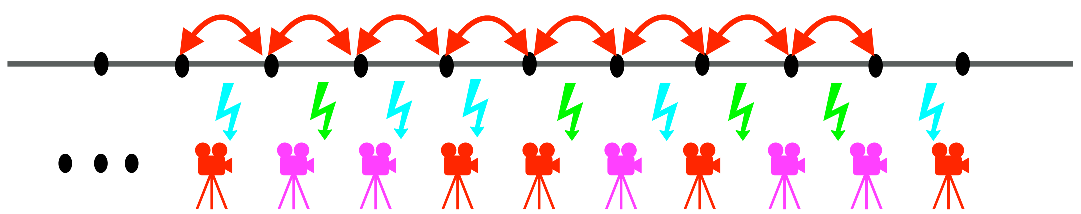
A complexity transition :

Volume law versus O(1) entanglement entropy

What is the MITP field theory ?

—> recent answer for monitored free fermions

[Fava, Piroli, D.B., Nahum]



— 1D free fermions (hoping) + monitoring, say $\gamma_j\gamma_{j+1}$

$$\rho_t \to \rho_t = \frac{\sigma_t}{\mathrm{tr}\sigma_t} \quad \text{with proba} \quad \mathrm{tr}\sigma_t$$

— Aim at computing moments of the density matrix => « replicas trick »

$$\mathbb{E}[\rho_t^{\otimes k}] = \mathbb{E}'[(\mathrm{tr}\sigma_t)\,(\tfrac{\sigma_t}{\mathrm{tr}\sigma_t})^{\otimes k}] = \mathbb{E}'[(\mathrm{tr}\sigma_t)^{N-k}\sigma_t^{\otimes k}]|_{N\to 1}$$

— Time evolution of σ_t is linear
After averaging => a spin chain with 'non-hermitian' hamiltonians.

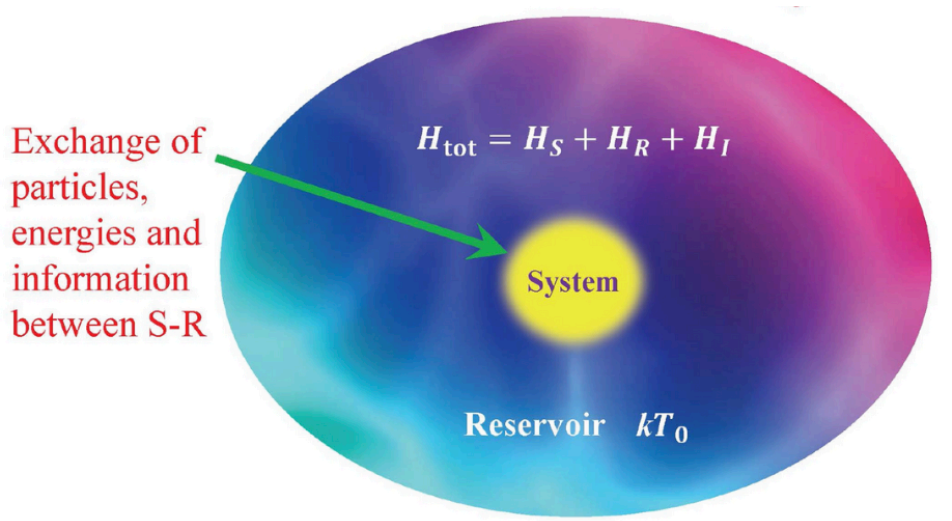
— Large distance / long time effective theory
==> Sigma models on SO(N), N->1

$$\mathcal{S} = \frac{1}{2g}\int \mathrm{d}x\mathrm{d}t\ \mathrm{Tr}\,(\partial_\mu Q)^T(\partial_\mu Q) + (\ldots),$$

for Q expect. of bilinear in fermions

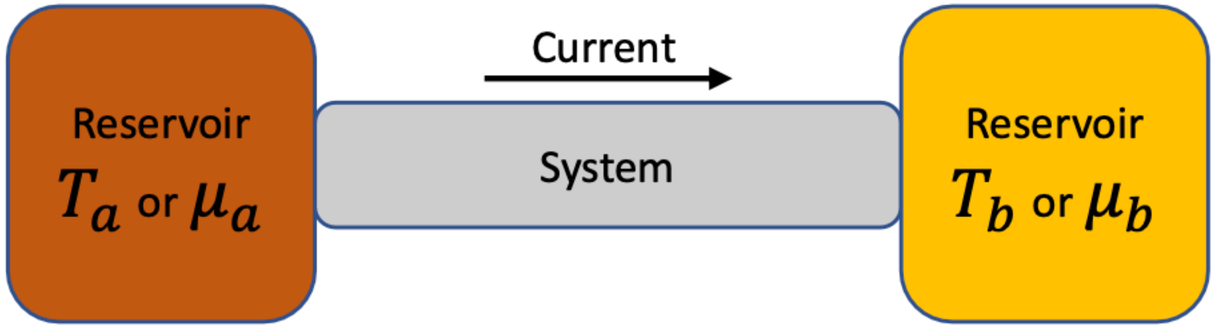
RG analysis, beta function => 1+1D marginally irrelevant (disentanglented+log-correction, fixed pint at higher D)

Balades around Quantum Stochastic Processes



III- « Extrinsic » : Transport & fluctuations in mesoscopic systems & QSSEP

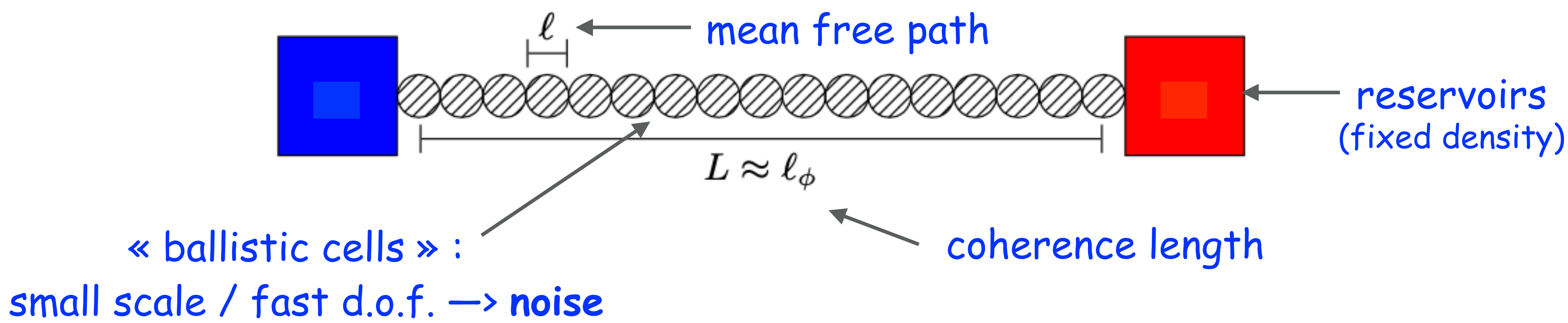
noisy/stochastic quantum dynamics (versus open quantum dynamics)
quantum diffusive systems
(classical) macroscopic fluctuations theory
excursion in probability theory (large deviation theory, Brownian motion & SDE…)



Fluctuating mesoscopic physics ? Quantum SSEP

(different route than RQC)

— « **Mesoscopic** » (diffusive + coherent) systems out-of-equilibrium



- Transport and its fluctuations (cf. classical Macroscopic Fluctuation Theory).
- Fluctuations of quantum coherent effects at mesoscopic scales & out-of-equilibrium.

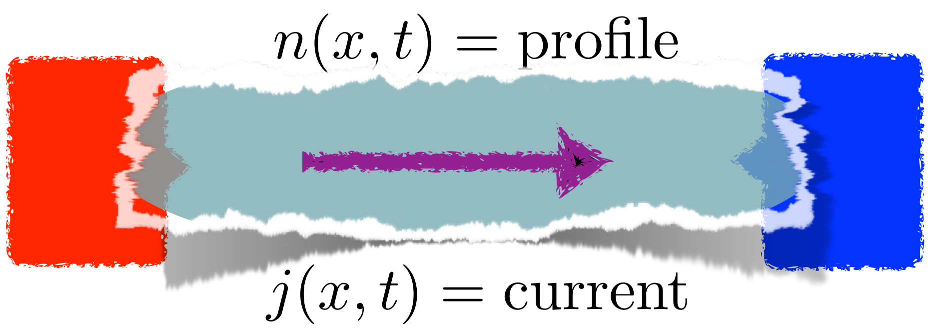
— **Is there universality in the fluctuations of quantum coherent effects in out-of-equilibrium diffusive (mesoscopic) many-body systems ?...**

The (classical) Macroscopic Fluctuation Theory

[Bertini, Sole, Gabrielli, Jona-Lasinio, Landim, …]

— MFT describes **fluctuations in out-of-equilibrium « classical » systems**

—> Statistics on profiles, currents, transport, and their fluctuations…



MFT : an effective theory via a noisy Fourier-Fick's law

Large deviation functions

$$\mathbb{P}\text{rob}\Big[\text{profile} = n(\cdot)\Big] \asymp e^{-(L/a_{uv})\,F[n(\cdot)]}$$

← « analogue » of free energy out-of-equilibrium (non-local)

—> Fluctuations are controlled by 1/L.

Intermezzo : large deviation « theory »

— **Large deviation : basics**

$$\mathbb{P}[X \in [x, x+dx]] \asymp e^{-I(x)/\epsilon}\, dx$$

—> cf. blackboard...

— **Large deviation & thermodynamics : « MFT at equilibrium »**

$$\mathbb{P}[n \neq \bar{n} \text{ in } v \ll V] \asymp e^{-v\, I(n)} \quad ,$$

—> cf. blackboard...

— **(Maybe?) Large aviation & small noise SDE**

—> cf. blackboard (if...) ...

The (classical) Macroscopic Fluctuation Theory

[Bertini, Sole, Gabrielli, Jona-Lasinio, Landim, …]

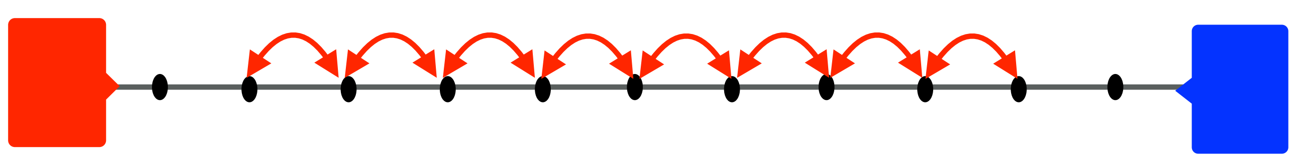
— MFT describes fluctuations in out-of-equilibrium « classical » systems

—> Statistics on profiles, currents, transport, and their fluctuations

—> An effective theory via a noisy Fourier-Fick's law

— **MFT emerged from studies of (stochastic) lattice models…**

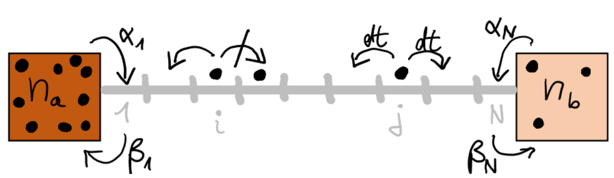
[Kipnis, Landim, Liggett, Spohn, Derrida, Mallick, Evans, et al …]



e.g. SSEP (Markov chain)

- independent random walkers

- symmetric simple exclusion process

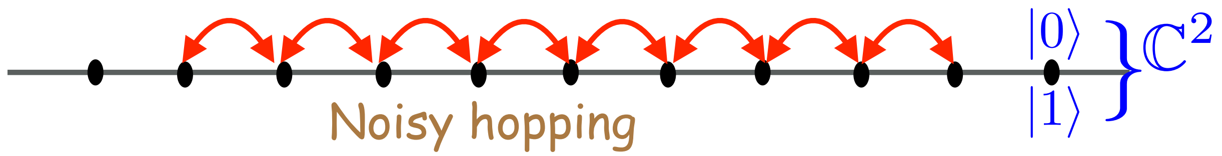


$$\begin{cases} \mathbb{M}_{\rm ssep}(|\emptyset\emptyset|) &= 0\,, \\ \mathbb{M}_{\rm ssep}(|\bullet\emptyset|) &= -|\bullet\emptyset| + |\emptyset\bullet|\,, \\ \mathbb{M}_{\rm ssep}(|\emptyset\bullet|) &= -|\emptyset\bullet| + |\bullet\emptyset|\,, \\ \mathbb{M}_{\rm ssep}(|\bullet\bullet|) &= 0\,. \end{cases}$$

— **Diffusive systems**

Quantum SSEP as quantum stochastic dynamics :

— QSSEP= **noisy hopping** model



$$dH_t = \sqrt{D}\,\sum_j \big(c^\dagger_{j+1} c_j\, dW^j_t + c^\dagger_j c_{j+1}\, d\overline{W^j_t}\big)$$

+ boundary terms…
+ injection/extraction…

« Exclusion » = Pauli principle

— Quantum stochastic dynamics and open system system dynamics

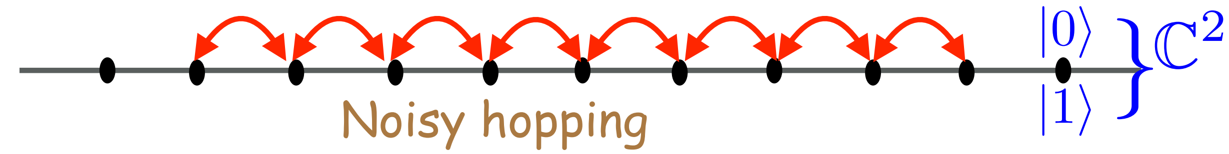
- random unitaires and CP map
- stochastic hamiltonian flows and Lindbladian
- moments and replicas
- etc ?…

—> cf. blackboard…

The Quantum SSEP (basics) :

[D.B.-T.Jin, 2019]

— QSSEP :



$$dH_t = \sqrt{D}\,\sum_j \,(c^\dagger_{j+1}c_j\,dW^j_t + c^\dagger_j c_{j+1}\,d\overline{W^j_t})$$

+ boundary terms…
+ injection/extraction…

—> Quadratic but **noisy** model…

— **Stochastic many-body quantum system** :

—> Stochastic process on « coherences »

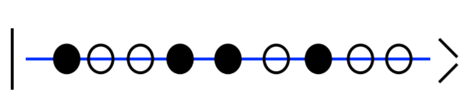
$$G_{ij} = \langle c^\dagger_j c_i\rangle_t = \mathrm{Tr}(\rho_t\, c^\dagger_j c_i)$$

$$G_{t+dt} = e^{-idh_t}G_t e^{+idh_t}$$

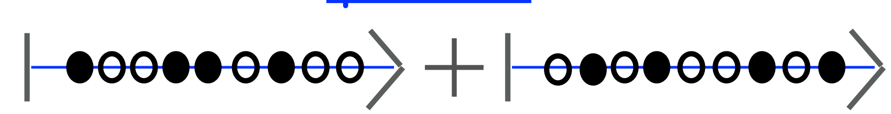
+ « boundary terms » (dh = one-particle hamiltonian)

— Q-SSEP includes SSEP (in mean) but codes for **quantum coherent effects.**

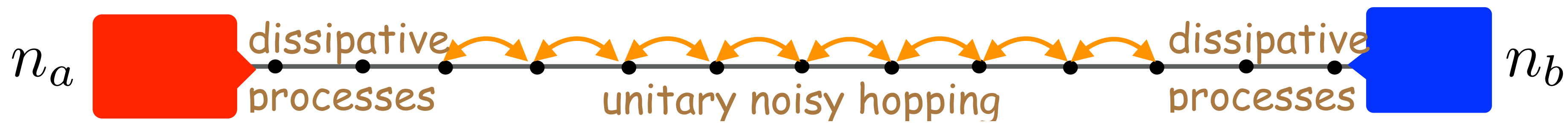
classical



quantum



Non-equilibrium coherent fluctuations

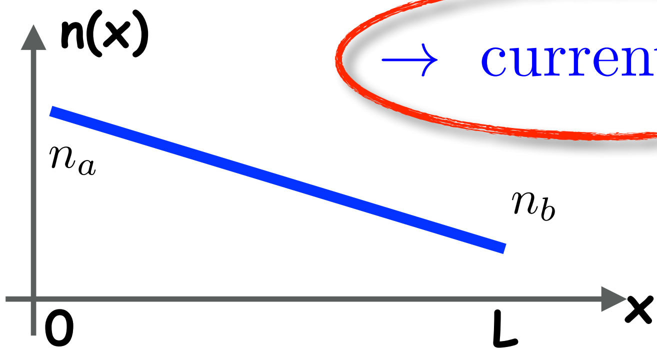


— **Steady mean profile** —> out-of-equilibrium:

$$[n_j] := \mathbb{E}[\langle c_j^\dagger c_j \rangle] = n_a + x(n_b - n_a)$$

(x=j/N, at large system size L=Na)

$\mathbb{E}[G_{ij}] = 0$ —> decoherence (in mean)



$\rightarrow$ current $j \neq 0$

— **Fluctuations & coherences** —> Steady statistics of (coherent) fluctuations.

$$\mathbb{E}[\,|G_{ij}|^2\,] = \frac{1}{N}(\Delta n)^2 x(1-y) + O(N^{-2}) \quad \text{for } G_{ij} = \langle c_j^\dagger c_i \rangle \quad (x = i/N < y = j/N)$$

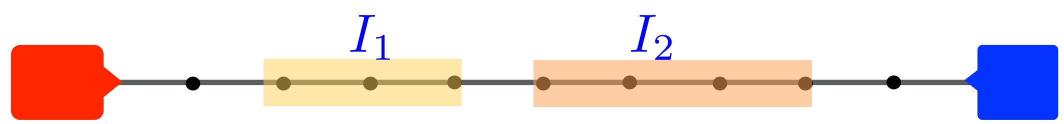
—> long range (multi-point) coherent correlations.

→ **Sub-leading** (in system size) **fluctuating coherences** (beyond mean decoherence)
… with some (hidden) patterns (—> free probability & large deviation…)

Coherences & Fluctuations (in Q-SSEP & more):

— **Transport (diffusive) is classical, at leading order…**

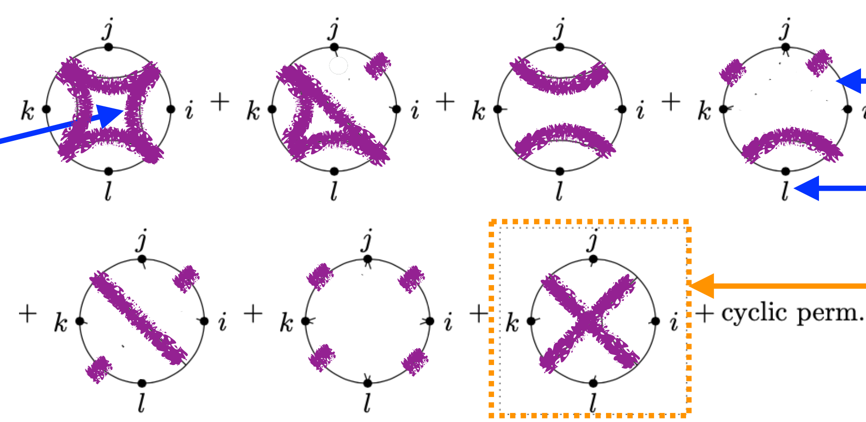
— « **Volume law** » for mutual information in mesoscopic systems
(contrary to systems at equilibrium or for ballistic systems : **long range correlations**)



$$I^{(q)}(I_1;I_2) := S^{(q)}_{I_1} + S^{(q)}_{I_2} - S^{(q)}_{I_1\cup I_2} \quad \text{with} \quad S^{(q)}_I = (1-q)^{-1}\,\mathbb{E}[\,\log \mathrm{Tr}(\rho_I^q)\,]$$

— « **Long-range correlations** » of coherence fluctuations & large deviations.

$$\mathbb{E}[G_{il}G_{lk}G_{kj}G_{ji}] =$$



cumulants
(partition of edges)

coherence G(ij)

point i,j,k,l

negligeable !

—> Emergence of **free probability** in noisy mesoscopic systems.

— Conjectural « **Universality** » in fluctuations of mesoscopic systems ???

→ Towards a « Quantum Mesoscopic Fluctuation Theory »


The End